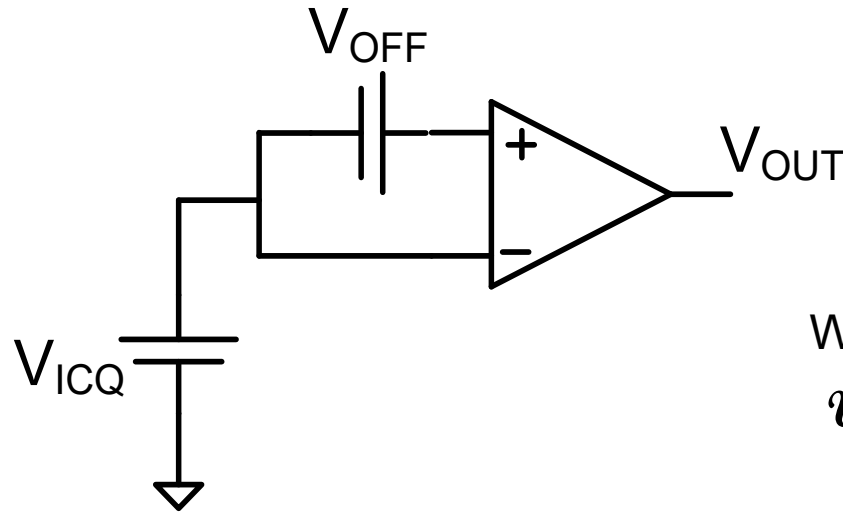


# EE 435

## Lecture 22

### Offset Voltages

# Offset Voltage



When operating linearly

$$v_{OUT} = A_D v_D + A_C v_C$$

**Definition:** The input-referred offset voltage is the differential dc input voltage that must be applied to obtain the desired output when  $V_{ic}$  is the quiescent common-mode input voltage.

$V_{OFF}$  is usually related to the output offset voltage by the expression

$$V_{OFF} = \frac{V_{OUTOFF}}{A_D}$$

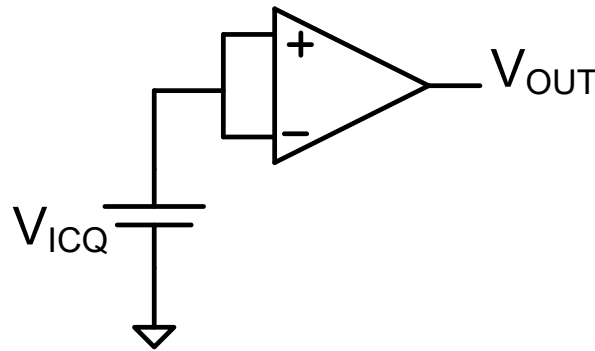
$V_{OFF}$  is dependent upon  $V_{ICQ}$  although this dependence is usually quite weak and often not specified

$V_{OFF}$  almost always large enough to force op amp out of linear mode for good op amps if used open loop

# Offset Voltage

Two types of offset voltage:

- Systematic Offset Voltage
- Random Offset Voltage

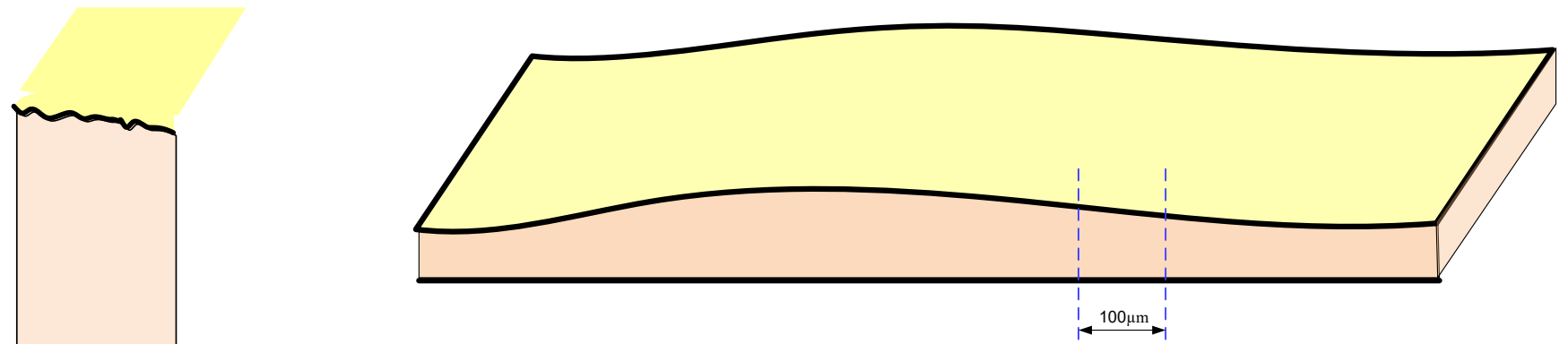


After fabrication it is impossible (difficult) to distinguish between the systematic offset and the random offset in any individual op amp

Measurements of offset voltages for a large number of devices will provide mechanism for identifying systematic offset and statistical characteristics of the random offset voltage

# Gradient and Local Random Effect

1



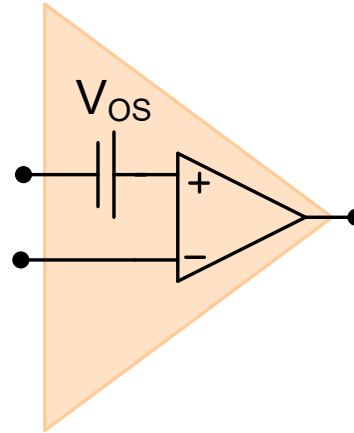
Gradient Effects : Locally Appear Linear

- Magnitude and Direction of Gradients are random
- Highly Correlated over Short Distances

Local Random Effects :  
Vary Locally With No  
Correlation

- **Both Contribute to Offset**
- **Both are random variables**
- **If Not Managed, Both Can Cause Large Offsets**
- **Strategies for minimizing their effects are different**
- **Will refer to the local random effects as “random” and the random gradient effects as “gradient” effects**

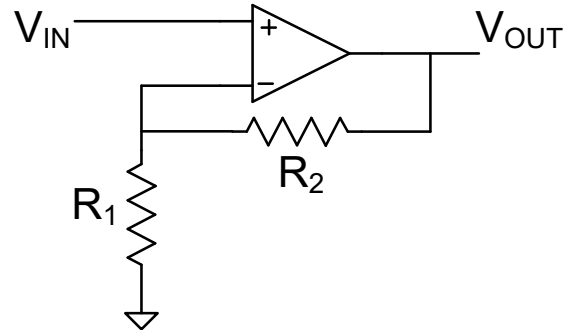
# Offset Voltage



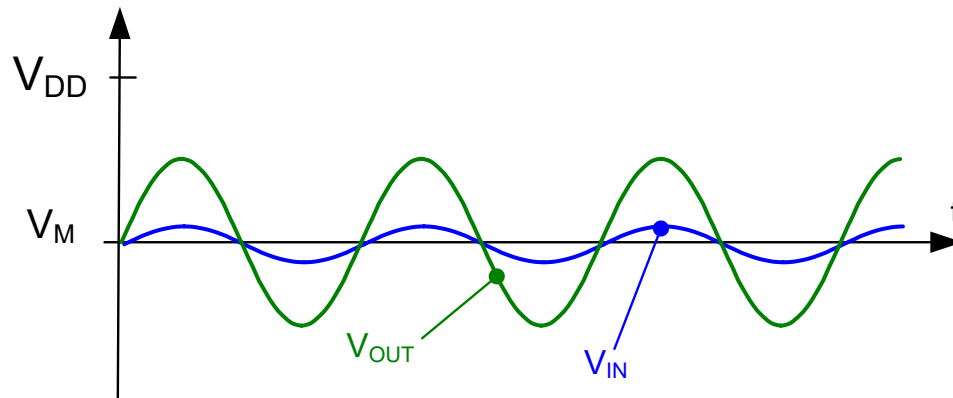
Can be modeled as a dc voltage source in series with the input

# Offset Voltage

Effects of Offset Voltage - an example

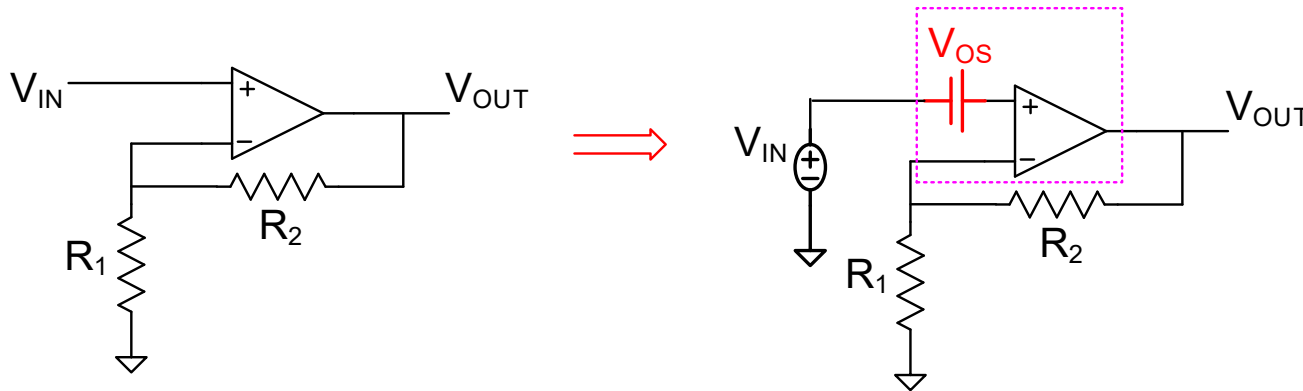


Desired I/O relationship



# Offset Voltage

Effects of Offset Voltage - an example



By Superposition:

$$V_{OUT} = \left(1 + \frac{R_2}{R_1}\right) V_{IN} + \left(1 + \frac{R_2}{R_1}\right) V_{OS}$$

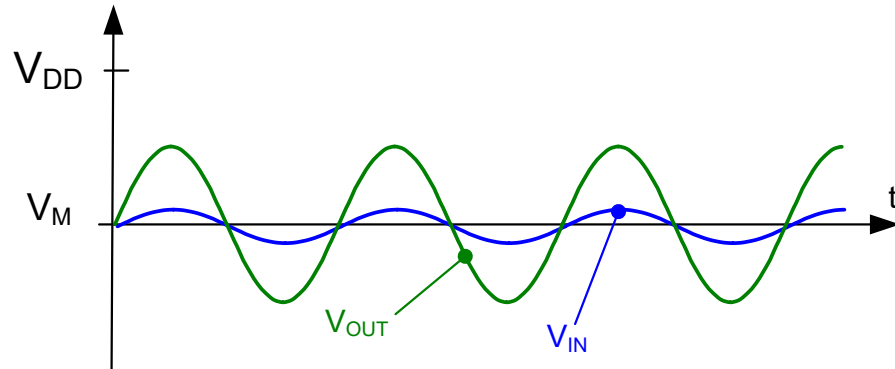
In this feedback circuit,  $V_{OS}$  multiplied by same gain as  $V_{IN}$

Particularly problematic if  $V_{IN}$  is small (so FB gain is then usually large)

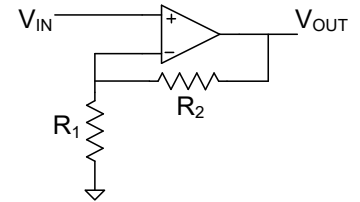
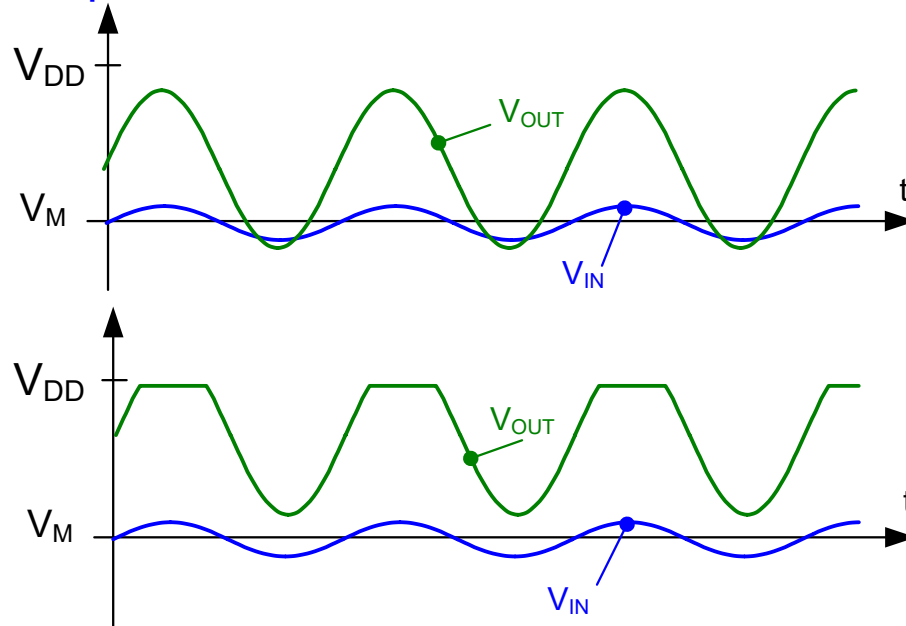
# Offset Voltage

Effects of Offset Voltage - an example

Desired I/O relationship

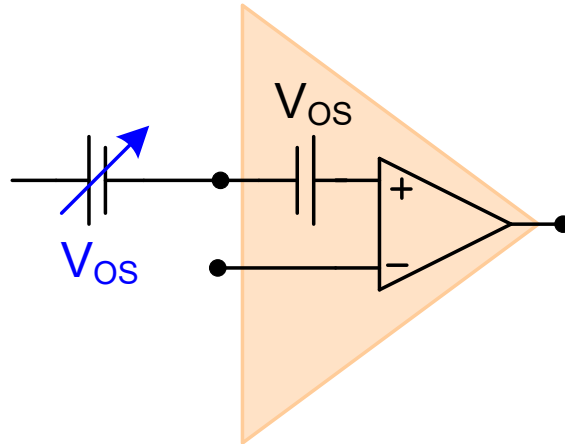


Actual I/O relationship due to offset





# Offset Voltage



Effects can be reduced or eliminated by adding equal amplitude opposite DC signal (many ways to do this)

Widely used in offset-critical applications

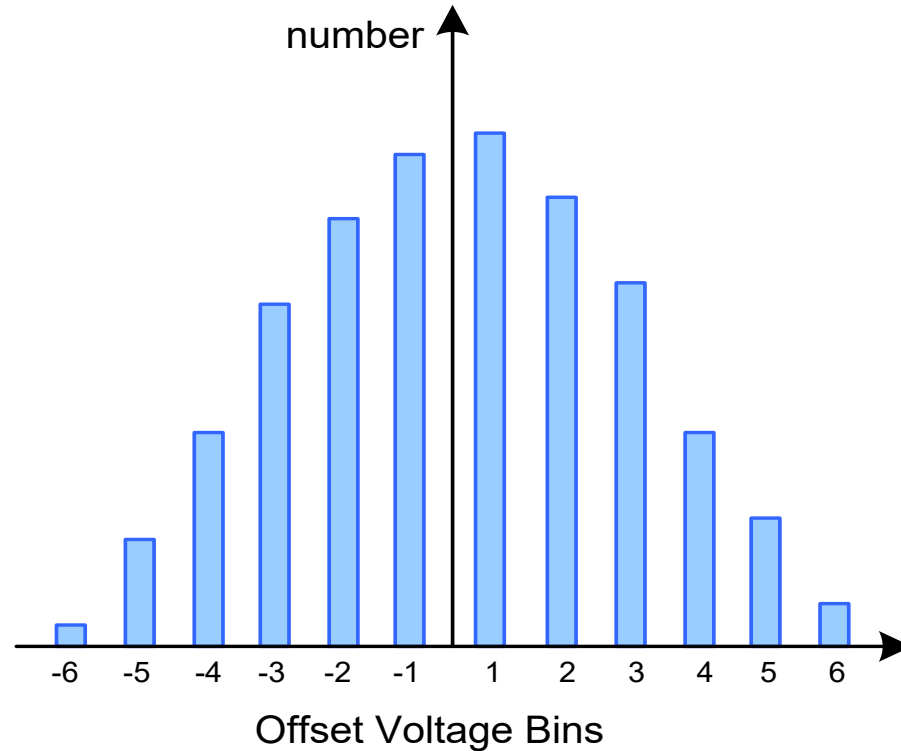
Comes at considerable effort and expense for low offset

**Prefer to have designer make  $V_{OS}$  small in the first place**

# Effects of Offset Voltage

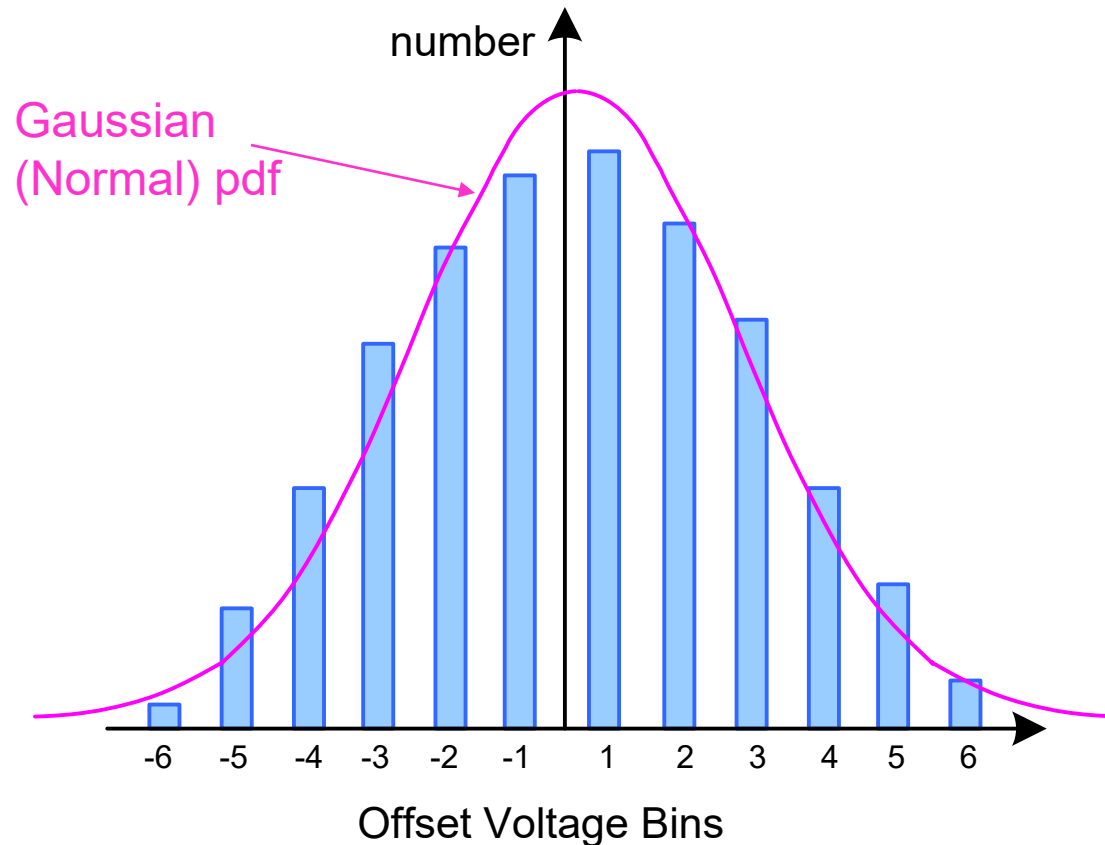
- Deviations in performance will change from one instantiation to another due to the random component of the offset
- Particularly problematic in circuits with high FB gain
- A major problem in many other applications
- Not of concern in many applications as well

# Random Offset Voltage Distribution



Typical histogram of random offset voltage (binned) after fabrication

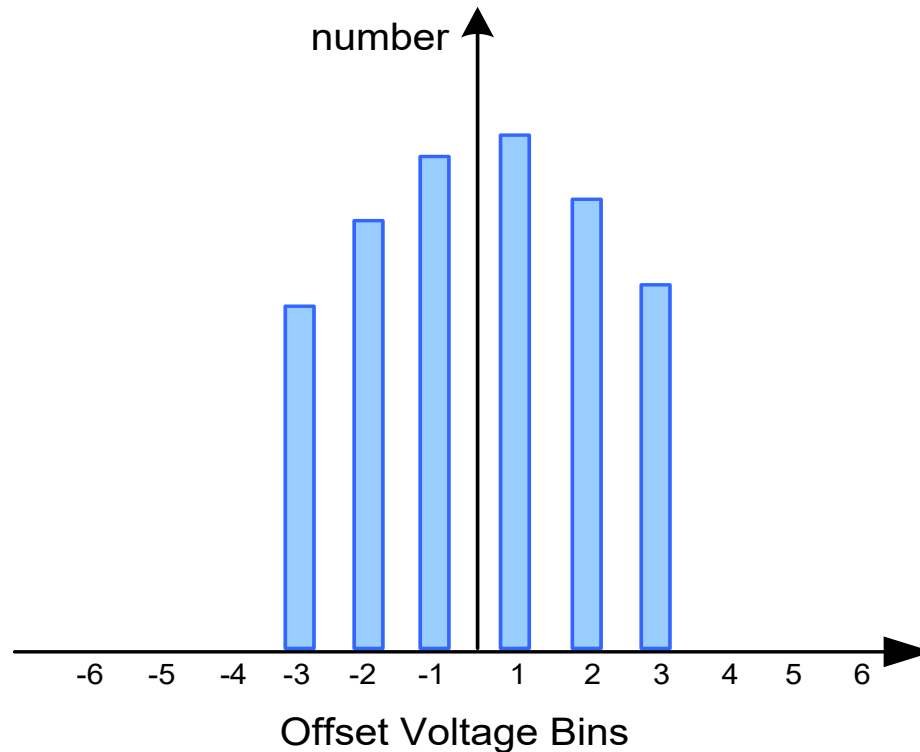
# Random Offset Voltage Distribution



Typical histogram of offset voltage (binned) after fabrication

Mean is nearly 0 (mean is actually the systematic offset voltage)

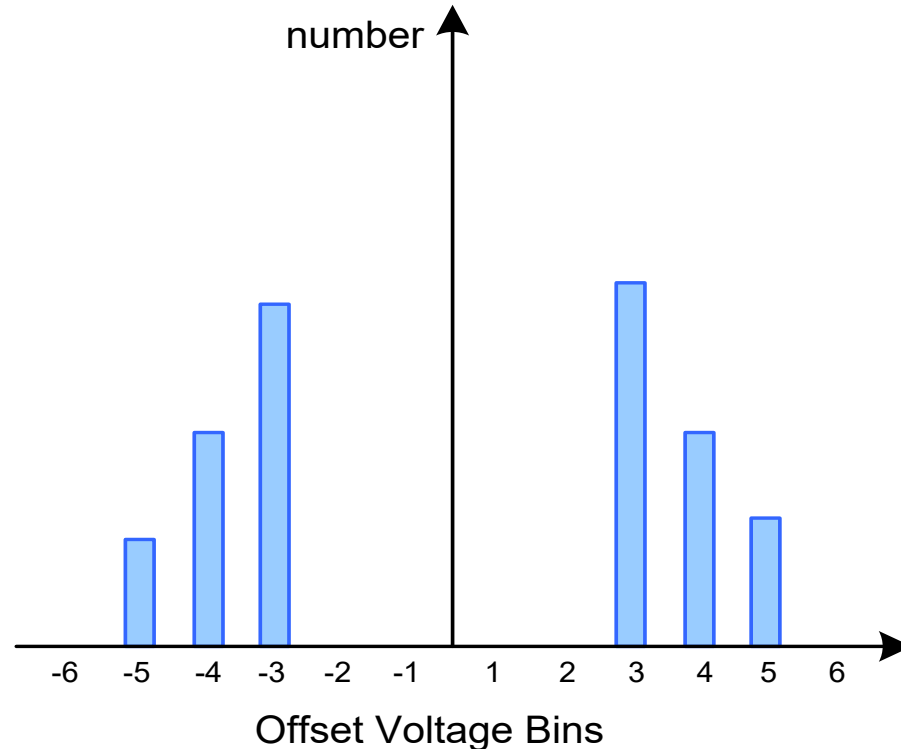
# Random Offset Voltage Distribution



Typical histogram of offset voltage (binned) in shipped parts

Extreme offset parts have been sifted at test

# Random Offset Voltage Distribution



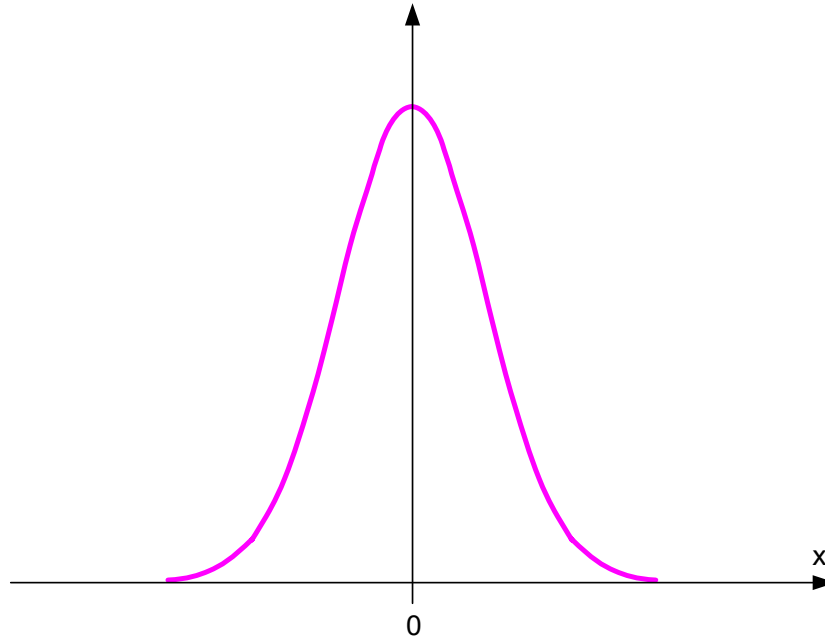
Typical histogram of offset voltage (binned) in shipped parts

Low-offset parts sometimes sold at a premium

Extreme offset parts have been sifted at test

# Random Offset Voltage Distribution

Pdf of zero-mean Gaussian distribution



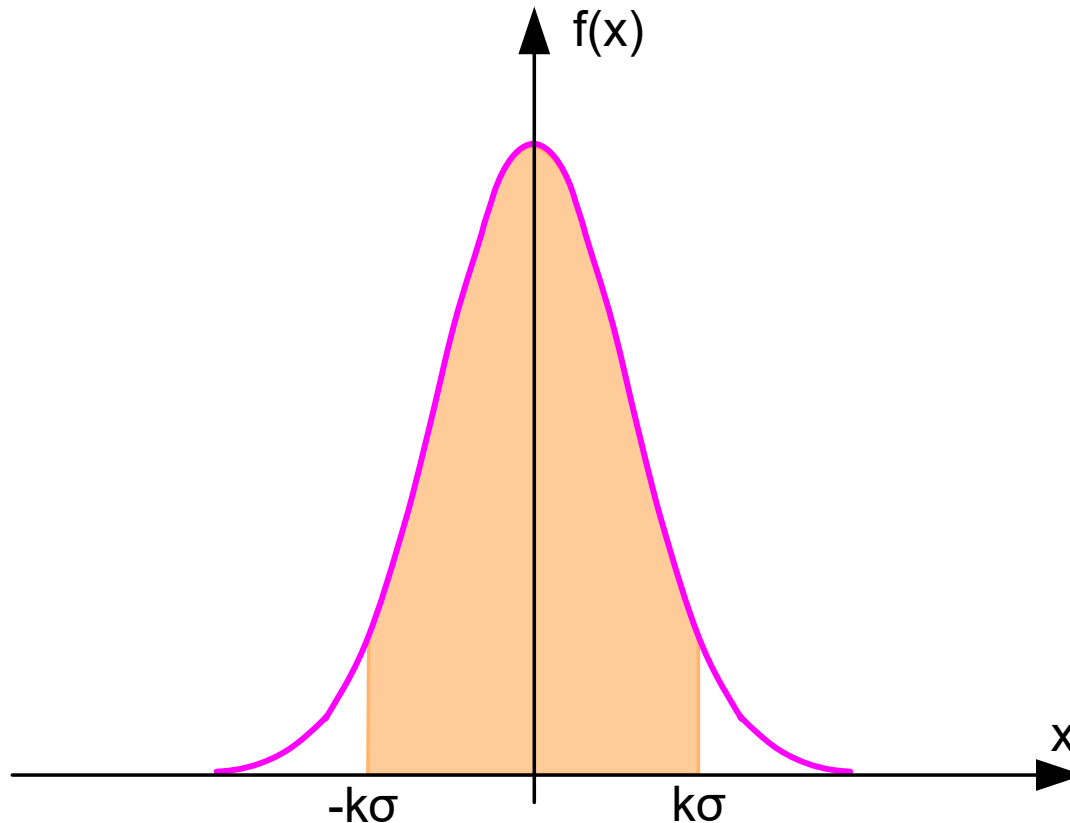
Characterized by its standard deviation  $\sigma$  or variance  $\sigma^2$

Offset voltage often specified as the  $1\sigma$  or  $3\sigma$  value (though authors may neglect to indicate which)

For catalog parts, often specified as the worst-case value after sorted

# Random Offset Voltage Distribution

Pdf of zero-mean Gaussian distribution



Percent between:	$\pm\sigma$	68.3%
	$\pm 2\sigma$	95.5%
	$\pm 3\sigma$	99.73%



# Offset Voltage



TL082-N

www.ti.com

SNOSBW5C – APRIL 1998 – REVISED APRIL 2013

## TL082 Wide Bandwidth Dual JFET Input Operational Amplifier

Check for Samples: [TL082-N](#)

### FEATURES

- Internally Trimmed Offset Voltage: 15 mV
- Low Input Bias Current: 50 pA
- Low Input Noise Voltage: 16nV/√Hz
- Low Input Noise Current: 0.01 pA/√Hz
- Wide Gain Bandwidth: 4 MHz
- High Slew Rate: 13 V/μs
- Low Supply Current: 3.6 mA
- High Input Impedance: 10<sup>12</sup>Ω
- Low Total Harmonic Distortion: ≤0.02%
- Low 1/f Noise Corner: 50 Hz
- Fast Settling Time to 0.01%: 2 μs

### DESCRIPTION

These devices are low cost, high speed, dual JFET input operational amplifiers with an internally trimmed input offset voltage ( BI-FET II™ technology). They require low supply current yet maintain a large gain bandwidth product and fast slew rate. In addition, well matched high voltage JFET input devices provide very low input bias and offset currents. The TL082 is pin compatible with the standard LM1558 allowing designers to immediately upgrade the overall performance of existing LM1558 and most LM358 designs.

These amplifiers may be used in applications such as high speed integrators, fast D/A converters, sample and hold circuits and many other circuits requiring low

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### DC Electrical Characteristics <sup>(1)</sup>

Symbol	Parameter	Conditions	TL082C			Units
			Min	Typ	Max	
V <sub>OS</sub>	Input Offset Voltage	R <sub>S</sub> = 10 kΩ, T <sub>A</sub> = 25°C		5	15	mV
		Over Temperature			20	mV

Sifted at test if  $|V_{OFF}| > 15\text{mV}$

Guess  $3\sigma$  value of trimmed but non-culled population is 15 mV

# Offset Voltage

## LM741 Operational Amplifier

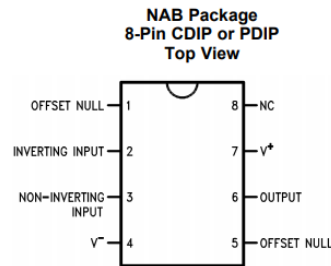
### 1 Features

- Overload Protection on the Input and Output
- No Latch-Up When the Common-Mode Range is Exceeded

### 2 Applications

### 3 Description

The LM741 series are general-purpose operational amplifiers which feature improved performance over industry standards like the LM709. They are direct, plug-in replacements for the 709C, LM201, MC1439, and 748 in most applications.



### 6.5 Electrical Characteristics, LM741<sup>(1)</sup>

PARAMETER	TEST CONDITIONS		MIN	TYP	MAX	UNIT
Input offset voltage	$R_S \leq 10 \text{ k}\Omega$	$T_A = 25^\circ\text{C}$		1	5	mV
		$T_{AMIN} \leq T_A \leq T_{AMAX}$			6	mV
Input offset voltage adjustment range	$T_A = 25^\circ\text{C}, V_S = \pm 20 \text{ V}$			$\pm 15$		mV

# Offset Voltage



[www.fairchildsemi.com](http://www.fairchildsemi.com)

## LM741 Single Operational Amplifier

### Features

- Short circuit protection
- Excellent temperature stability
- Internal frequency compensation
- High Input voltage range
- Null of offset

### Description

The LM741 series are general purpose operational amplifiers. It is intended for a wide range of analog applications. The high gain and wide range of operating voltage provide superior performance in integrator, summing amplifier, and general feedback applications.

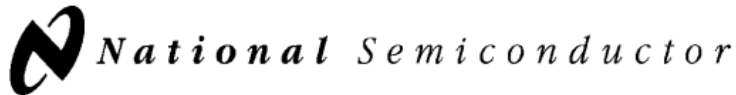


### Electrical Characteristics

( $V_{CC} = 15V$ ,  $V_{EE} = -15V$ ,  $T_A = 25^\circ C$ , unless otherwise specified)

Parameter	Symbol	Conditions	LM741C/LM741			Unit
			Min.	Typ.	Max.	
Input Offset Voltage	$V_{IO}$	$R_S \leq 10K\Omega$	-	2.0	6.0	mV
		$R_S \leq 50\Omega$	-	-	-	
Input Offset Voltage Adjustment Range	$V_{IO(R)}$	$V_{CC} = \pm 20V$	-	$\pm 15$	-	mV
Input Offset Current	$I_{IO}$	-	-	20	200	nA
Input Bias Current	$I_{BIAS}$	-	-	80	500	nA

# Offset Voltage



November 1994

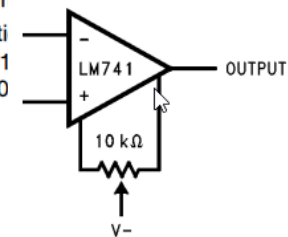
## LM741 Operational Amplifier

### General Description

The LM741 series are general purpose operational amplifiers which feature improved performance over industry standards like the LM709. They are direct, plug-in replacements for the 709C, LM201, MC1439 and 748 in most applications. The amplifiers offer many features which make their application nearly foolproof: overload protection on the input and

output, no latch-up when the  $V_{CC}$  exceeded, as well as freedom from The LM741C/LM741E are identical except that the LM741C/LM741E are guaranteed over a  $0^{\circ}\text{C}$  to  $+70^{\circ}\text{C}$  instead of  $-55^{\circ}\text{C}$  to  $+125^{\circ}\text{C}$ .

Offset Nulling Circuit

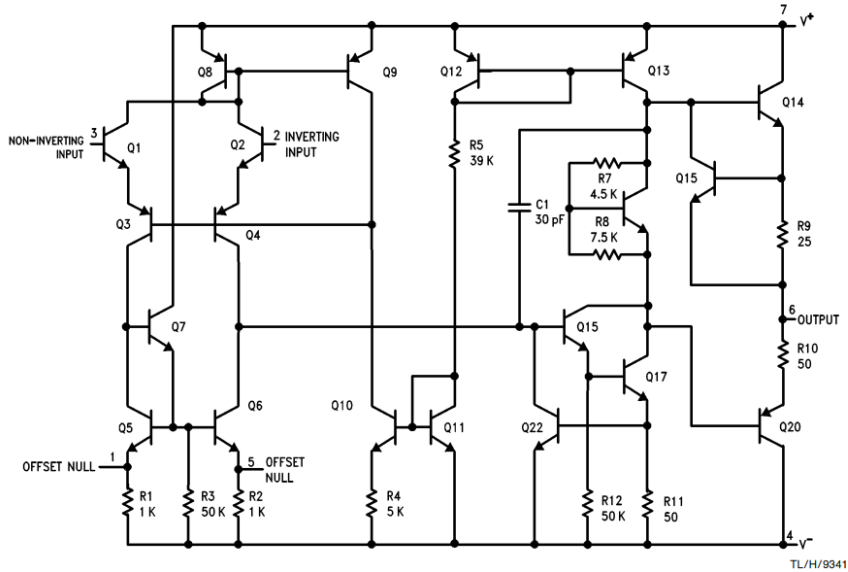


### Electrical Characteristics (Note 3)

Parameter	Conditions	LM741A/LM741E			LM741			LM741C			Units
		Min	Typ	Max	Min	Typ	Max	Min	Typ	Max	
Input Offset Voltage	$T_A = 25^{\circ}\text{C}$ $R_S \leq 10\text{ k}\Omega$ $R_S \leq 50\Omega$		0.8	3.0		1.0	5.0		2.0	6.0	mV mV
	$T_{AMIN} \leq T_A \leq T_{AMAX}$ $R_S \leq 50\Omega$ $R_S \leq 10\text{ k}\Omega$			4.0			6.0			7.5	mV mV
Average Input Offset Voltage Drift				15							$\mu\text{V}/^{\circ}\text{C}$

# Offset Voltage

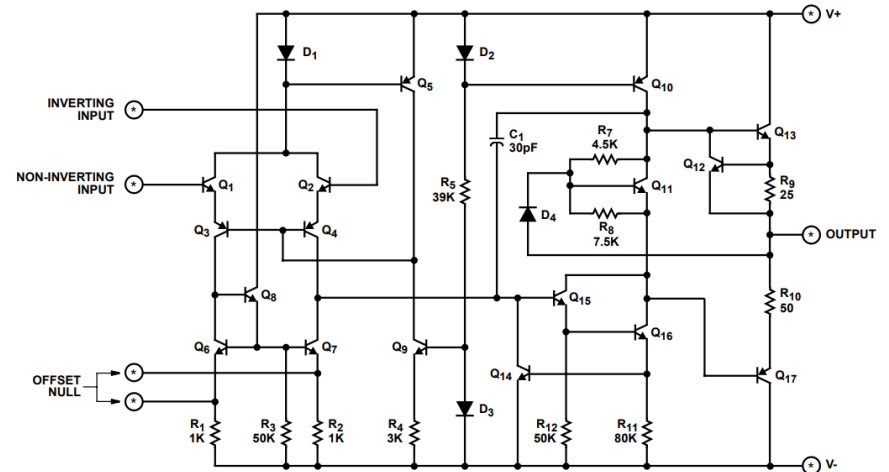
Schematic Diagram



National

Schematic Diagram (Notes 5, 6)

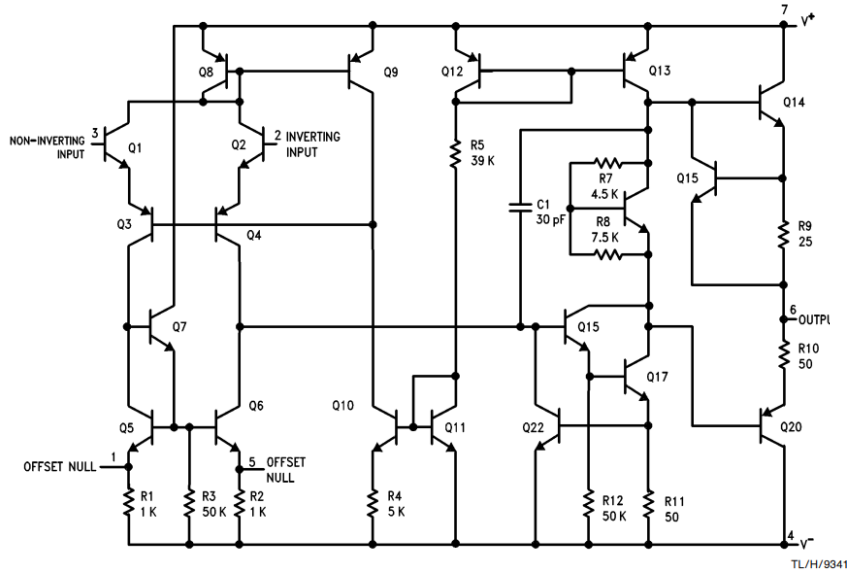
CA741C, CA741, LM741C, LM741 AND FOR EACH AMPLIFIER OF THE CA1458, CA1558, AND LM1458



Intersil

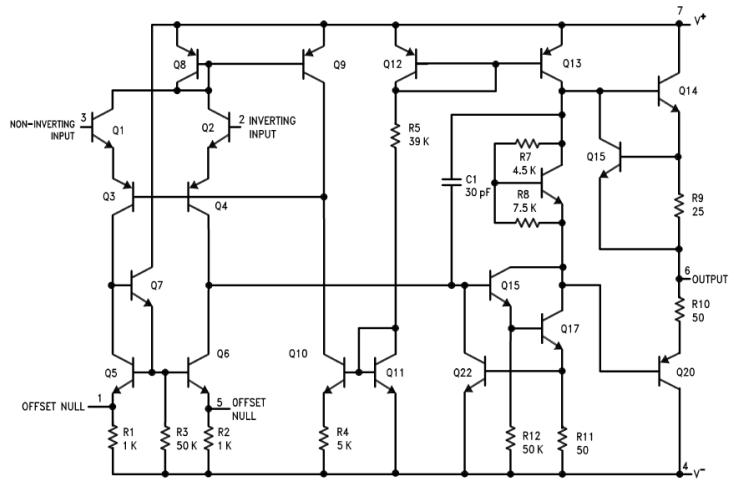
# Offset Voltage

Schematic Diagram



National

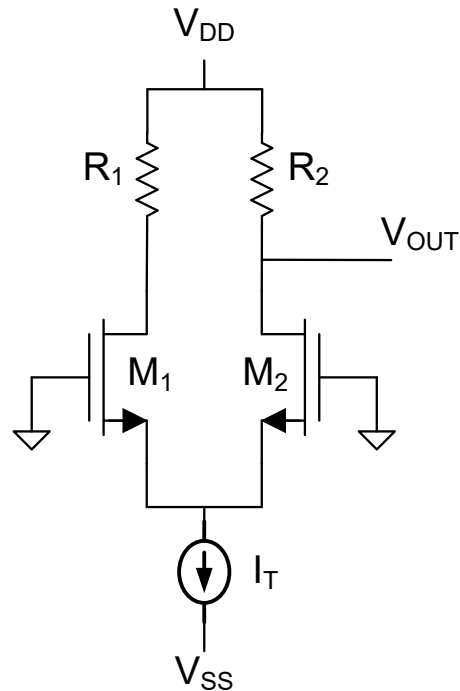
7.2 Functional Block Diagram



Texas Instruments

# Source of Random Offset Voltages

Consider as an example:



Ideally  $R_1=R_2=R$ ,  $M_1$  and  $M_2$  are matched

$$V_{OUT} = V_{DD} - \left(\frac{I_T}{2}\right)R$$

Assume this is the desired output voltage

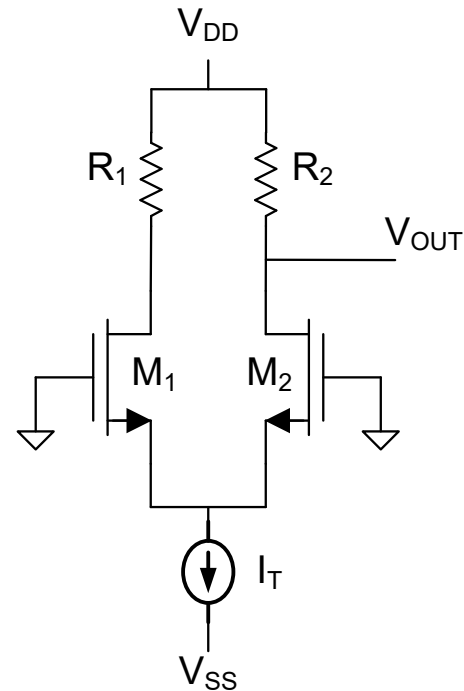
# Source of Random Offset Voltages

Consider as an example:

Ideally

$$R_1 = R_2 = R$$

$M_1$  and  $M_2$  matched



If everything ideal except  $R_2 = R + \Delta R$  (actually there will be mismatches between  $M_1$  and  $M_2$  also)

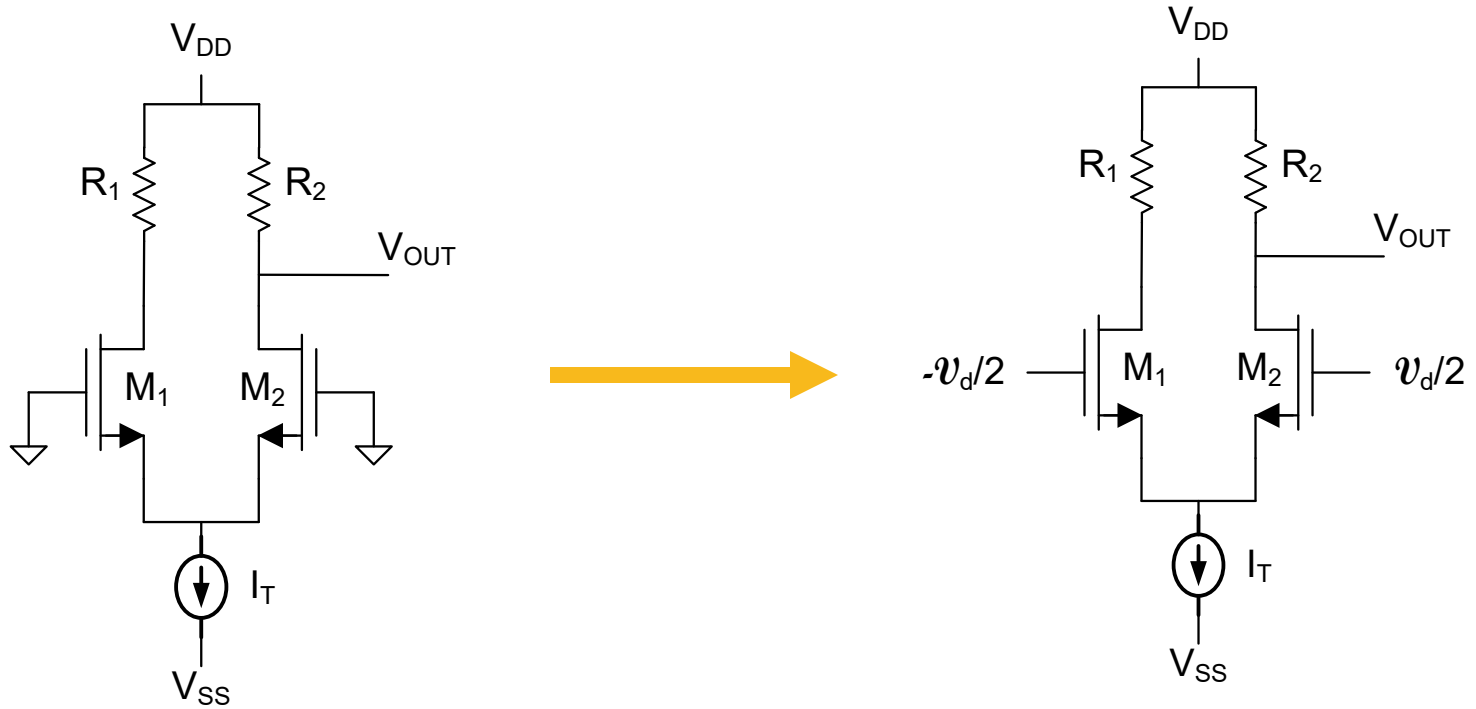
$$V_{OUT} = V_{DD} - \left( \frac{I_T}{2} \right) [R + \Delta R]$$

$$\Delta V_{OUT} = - \left( \frac{I_T}{2} \right) \Delta R$$



# Source of Random Offset Voltages

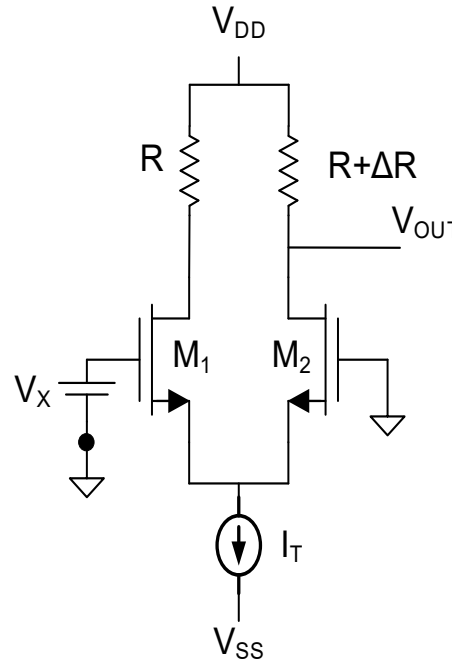
Consider as an example:



$$A_V = -\frac{g_m R}{2}$$

# Source of Random Offset Voltages

Determine the offset voltage – i.e. value of  $V_X$  needed to obtain desired output



$$A_V = \frac{g_m R}{2}$$

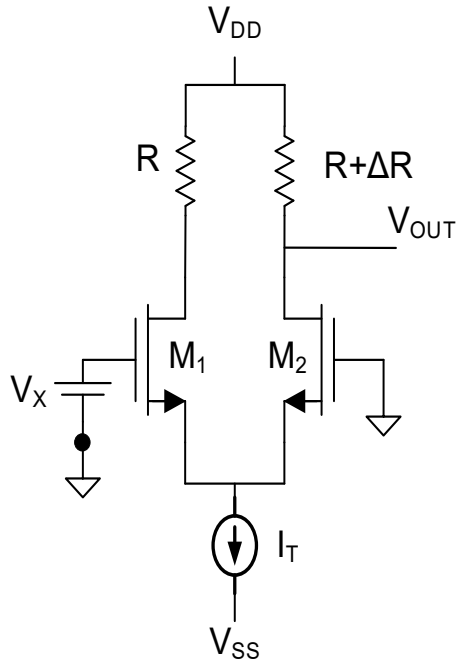
$$V_{OUT} = \left[ V_{DD} - \left( \frac{I_T}{2} \right) R \right] - \left( \frac{I_T}{2} \right) \Delta R + A_V V_X$$

To force  $V_{OUT}$  to its desired value of  $V_{OUT} = \left[ V_{DD} - \left( \frac{I_T}{2} \right) R \right]$

$$V_X = \frac{1}{A_V} \left( \frac{I_T}{2} \right) \Delta R$$

# Source of Random Offset Voltages

Determine the offset voltage – i.e. value of  $V_X$  needed to obtain desired output



$$A_V = \frac{g_m R}{2}$$

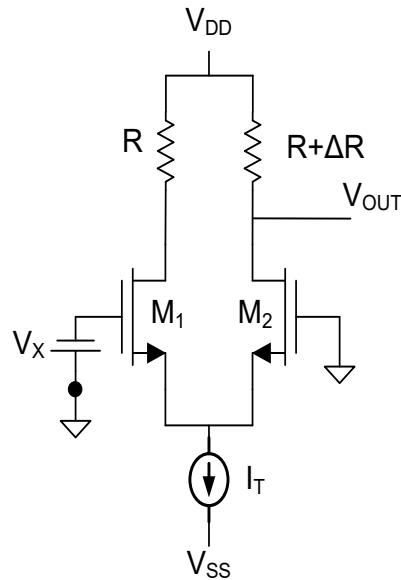
$$V_X = \frac{1}{A_V} \left( \frac{I_T}{2} \right) \Delta R$$

$$V_X = \frac{2}{g_m R} \left( \frac{I_T}{2} \right) \Delta R = \left( \frac{I_T}{g_m} \right) \frac{\Delta R}{R} = \left( \frac{I_T}{I_T / V_{EB}} \right) \frac{\Delta R}{R} = V_{EB} \frac{\Delta R}{R}$$


$$V_X = V_{EB} \frac{\Delta R}{R}$$

# Source of Random Offset Voltages

Determine the offset voltage – i.e. value of  $V_x$  needed to obtain desired output



$$A_V = \frac{g_m R}{2}$$

$$V_x = V_{EB} \frac{\Delta R}{R}$$

What can the designer do to reduce the offset voltage if the resistor value and statistics are fixed?

Reduce  $V_{EB}$

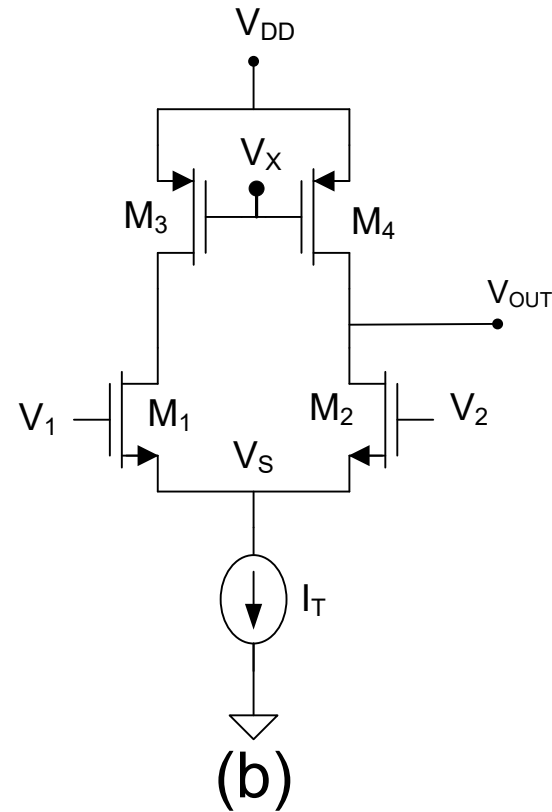
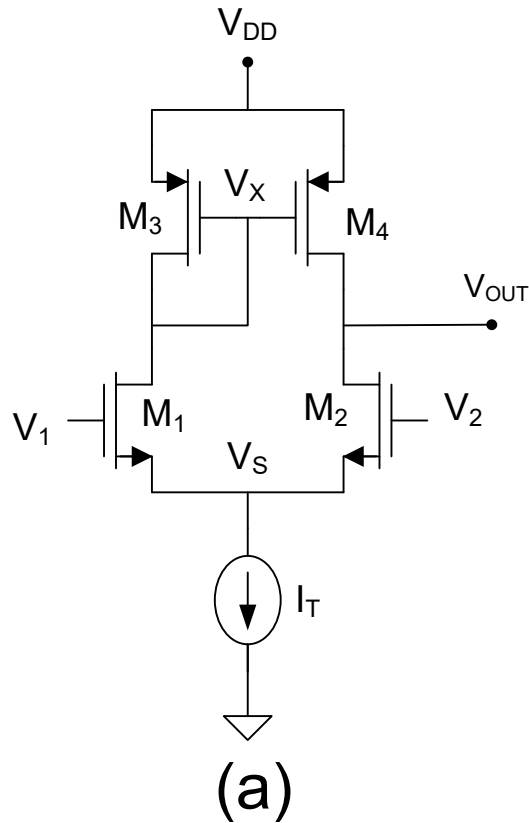
Will that affect the voltage gain?

$$A_V = \frac{g_m R}{2} = \frac{2 \frac{I_T}{2V_{EB}}}{2} R = \frac{1}{2V_{EB}} I_T R$$

Not if  $I_T$  is reduced by the same amount but that will affect signal swing and GB

# Source of Random Offset Voltages

The random offset voltage is almost entirely that of the input stage in most op amps



# Random Offset Voltage

- Due to random variations in process parameters and device dimensions
- Random offset is actually a random variable at the design level but deterministic after fabrication in any specific device
- Distribution naturally nearly Gaussian (could be un-naturally manipulated)

Has zero mean

Characterized by its standard deviation or variance

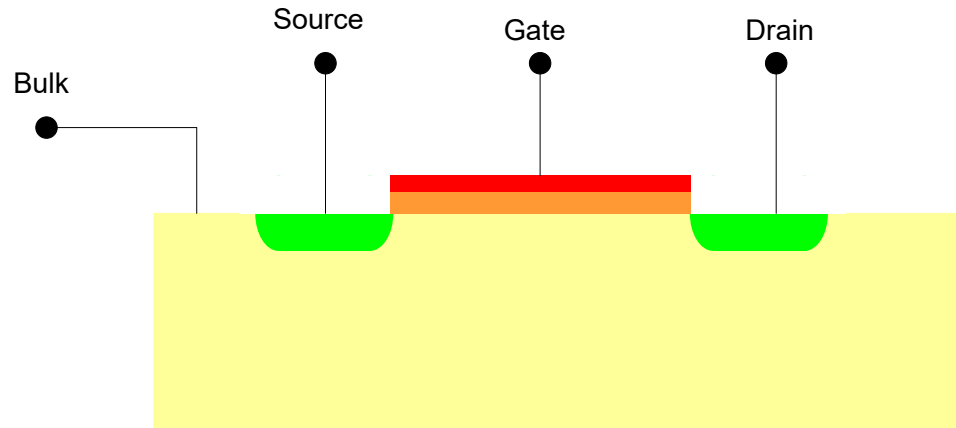
Often strongly layout dependent

Due to both local random variations and correlated gradient effects

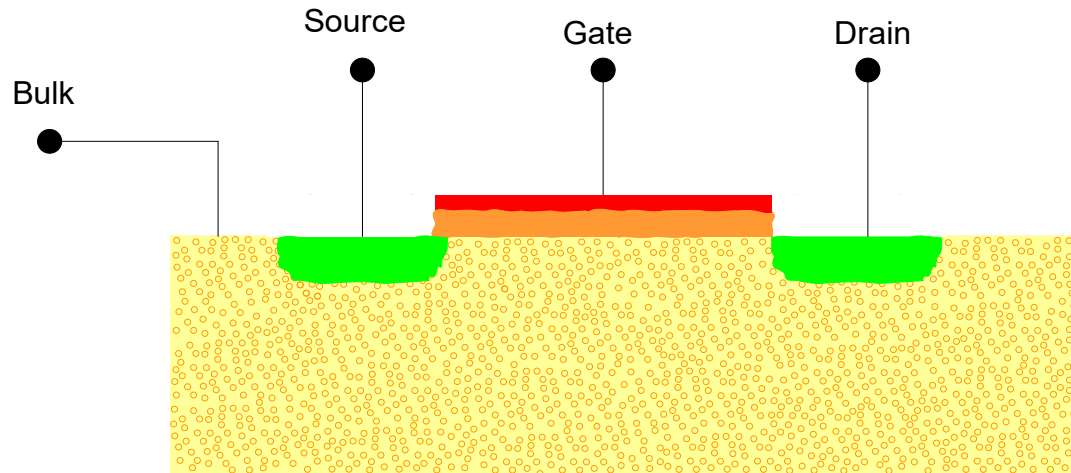
- Will consider both effects separately
- Gradient effects usually dominate if not managed
- Good methods exist for driving gradient effects to small levels

Notation: Henceforth, the term “Random Offset” will refer to that due to local random variations and gradient effects will be considered separately

# Offset Voltages due to Local Random Variations



n-channel MOSFET



n-channel MOSFET

Impurities vary randomly with position as do edges of gate, oxide and diffusions

Model and design parameters vary throughout channel and thus the corresponding equivalent lumped model parameters will vary from device to device

# Model Parameter Variation

Define  $p$  to be a process parameter that varies with lateral position throughout the region defined by the channel of the transistor.

Almost Theorem:

If  $p(x,y)$  varies throughout a two-dimensional region, then

$$p_{EQ} = \frac{1}{A} \int_A p(x,y) dx dy$$

Parameters such as  $V_T$ ,  $\mu$  and  $C_{OX}$  vary throughout a two-dimensional region

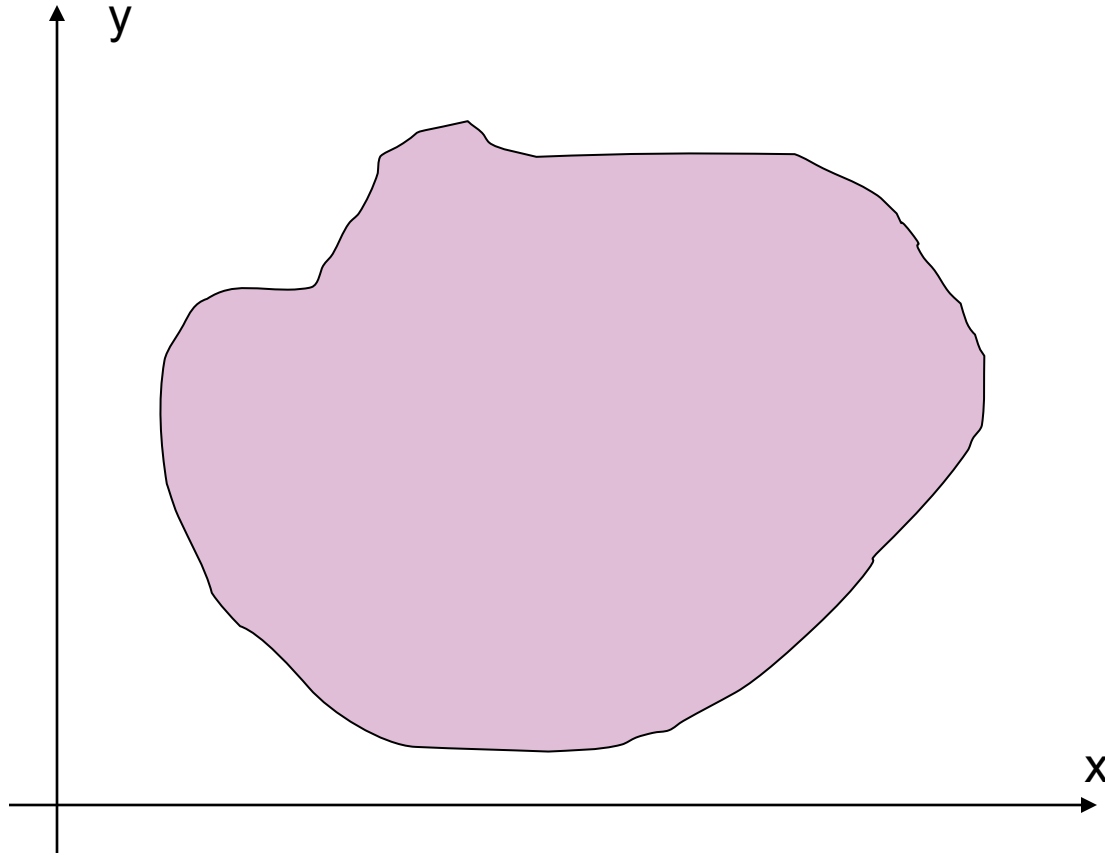
Local random variations introduce a random component in device model parameters which are uncorrelated but for ideally matched devices they are identically distributed

e.g.  $V_{TEQi} = V_{TN} + V_{TRi}$

$V_{TRi}$  and  $V_{TRj}$  due to local random variations are uncorrelated for  $i \neq j$  but if ideally matched they are identically distributed



# Model Parameter Variation



$$p_{EQ} = \frac{1}{A} \int_A p(x, y) dx dy$$

# Random Offset Voltages

The random offset associated with local random variations is due to mismatches in the four transistors, dominantly mismatches in the parameters  $\{V_T, \mu, C_{OX}, W$  and  $L\}$

The relative mismatch effects become more pronounced as devices become smaller

$$V_{Ti} = V_{TN} + V_{TRi}$$

$$C_{OXi} = C_{OXN} + C_{OXRi}$$

$$\mu_i = \mu_N + \mu_{Ri}$$

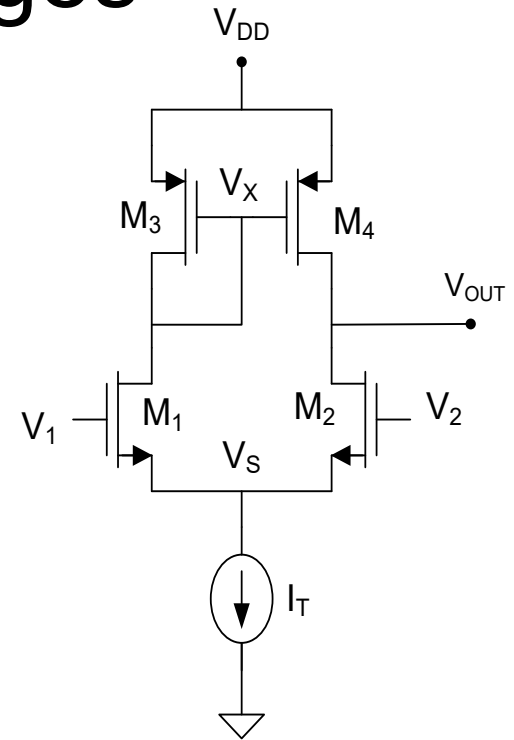
$$W_i = W_N + W_{Ri}$$

$$L_i = L_N + L_{Ri}$$

Each design and model parameter is comprised of a nominal part and a random component

It will be assumed that the random parts of each model parameter are uncorrelated but if ideally matched are identically distributed

(actually some small correlation in “model” parameters but will neglect in this course)



# Random Offset Voltages

$$V_{Ti} = V_{TN} + V_{TRi}$$

$$C_{OXi} = C_{OXN} + C_{OXRi}$$

$$\mu_i = \mu_N + \mu_{Ri}$$

$$W_i = W_N + W_{Ri}$$

$$L_i = L_N + L_{Ri}$$

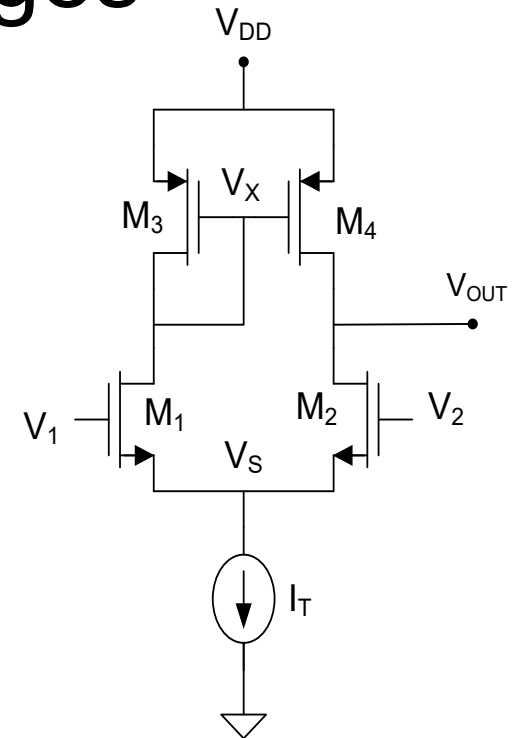
For each device, the device model is often expressed as

$$I_{Di} = \frac{(\mu_N + \mu_{Ri})(C_{OXN} + C_{OXRi})(W_N + W_{Ri})}{2(L_N + L_{Ri})} (V_{GSi} - (V_{TN} + V_{TRi}))^2 (1 + (\lambda_N + \lambda_{Ri})[V_{DS}])$$

Because of the random components of the parameters in every device, matching from the left-half circuit to the right half-circuit is not perfect

This mismatch introduces an offset voltage which is a random variable

For this 4-transistor op amp, there are 24 uncorrelated random variables



# Random Offset Voltages

From a straightforward but tedious analysis (involving the 24 random variables) it follows that:

$$\sigma_{V_{os}}^2 = 2 \left[ \frac{A_{V_{TO}n}^2}{W_n L_n} + \frac{\mu_p}{\mu_n} \frac{L_n}{W_n L_p^2} A_{V_{TO}p}^2 + \frac{V_{EBn}^2}{4} \left( \frac{1}{W_n L_n} A_{\mu_n}^2 + \frac{1}{W_p L_p} A_{\mu_p}^2 + A_{COX}^2 \left[ \frac{1}{W_n L_n} + \frac{1}{W_p L_p} \right] \right) \right. \\ \left. + 2A_L^2 \left[ \frac{1}{W_n L_n^2} + \frac{1}{W_p L_p^2} \right] + A_W^2 \left[ \frac{1}{L_n W_n^2} + \frac{1}{L_p W_p^2} \right] \right]$$

where the terms  $A_{V_{TO}}$ ,  $A_{\mu}$ ,  $A_{COX}$ ,  $A_L$ , and  $A_W$  are process parameters

$$A_{V_{TO}} \cong \begin{cases} 21\text{mV}\cdot\mu & \text{(n-ch)} \\ 25\text{mV}\cdot\mu & \text{(p-ch)} \end{cases}$$

Typical values for matching model parameters:

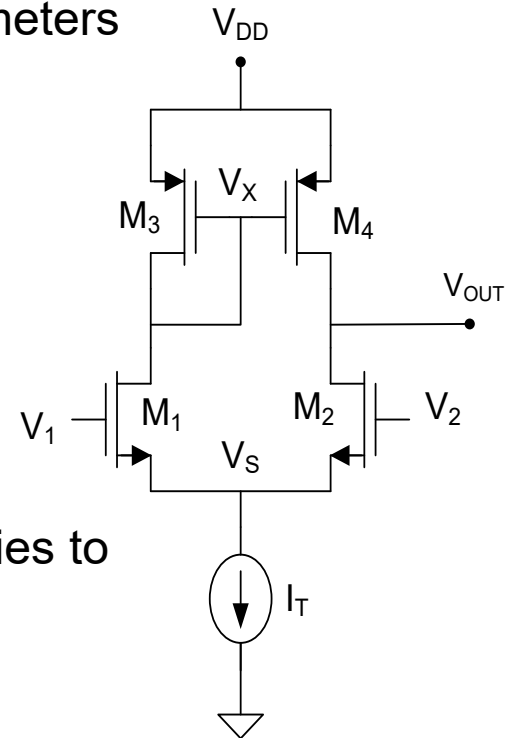
$$\sqrt{A_{\mu}^2 + A_{COX}^2} \cong \begin{cases} .016\mu & \text{(n-ch)} \\ .023\mu & \text{(p-ch)} \end{cases}$$

$$A_L = A_W \cong 0.017\mu^{3/2}$$

Usually the  $A_{V_{TO}}$  terms are dominant, thus the variance simplifies to

$$\sigma_{V_{os}}^2 \cong 2 \left[ \frac{A_{V_{TO}n}^2}{W_n L_n} + \frac{\mu_p}{\mu_n} \frac{L_n}{W_n L_p^2} A_{V_{TO}p}^2 \right]$$

(Remember this is due to local random variations)



# Random Offset Voltages

$$\sigma_{V_{os}}^2 \cong 2 \left[ \frac{A_{VTO n}^2}{W_n L_n} + \frac{\mu_p L_n}{\mu_n W_n L_p^2} A_{VTO p}^2 \right]$$

This expression has somewhat peculiar coefficients. The first term on the right is dependent upon the reciprocal of the area of the n-channel device but the corresponding coefficient on the second term on the right appears to depend upon the dimensions of both the n-channel and p-channel devices. But this can be rewritten as

$$\sigma_{V_{os}}^2 \cong 2 \left[ \frac{A_{VTO n}^2}{W_n L_n} + \left( \frac{V_{EB n}}{V_{EB p}} \right)^2 \frac{A_{VTO p}^2}{W_p L_p} \right]$$

The dependence of the variance on the area of the n-channel and p-channel devices is more apparent when written in this form.

# Random Offset Voltages

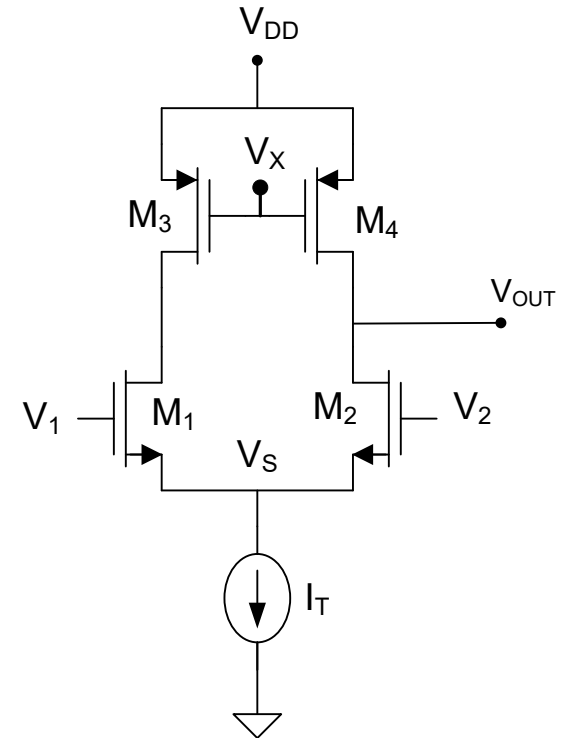
Correspondingly for differential output structure:

$$\sigma_{V_{os}}^2 = 2 \left[ \frac{A_{VTO n}^2}{W_n L_n} + \frac{\mu_p}{\mu_n} \frac{L_n}{W_n L_p^2} A_{VTO p}^2 + \frac{V_{EBn}^2}{4} \left( \frac{1}{W_n L_n} A_{\mu_n}^2 + \frac{1}{W_p L_p} A_{\mu_p}^2 + A_{COX}^2 \left[ \frac{1}{W_n L_n} + \frac{1}{W_p L_p} \right] \right) \right. \\ \left. + 2A_L^2 \left[ \frac{1}{W_n L_n^2} + \frac{1}{W_p L_p^2} \right] + A_w^2 \left[ \frac{1}{L_n W_n^2} + \frac{1}{L_p W_p^2} \right] \right]$$

which again simplifies to

$$\sigma_{V_{os}}^2 \cong 2 \left[ \frac{A_{VTO n}^2}{W_n L_n} + \frac{\mu_p}{\mu_n} \frac{L_n}{W_n L_p^2} A_{VTO p}^2 \right]$$

Note these offset voltage expressions are identical!



# Random Offset Voltages

Example: Determine the  $3\sigma$  value of the input offset voltage for The MOS differential amplifier if

a)  $M_1$  and  $M_3$  are minimum-sized and

b) the area of  $M_1$  and  $M_3$  are 100 times minimum size (with  $L_n=L_p$ )

$$\sigma_{V_{os}}^2 \cong 2 \left[ \frac{A_{VTO n}^2}{W_n L_n} + \frac{\mu_p L_n}{\mu_n W_n L_p^2} A_{VTO p}^2 \right]$$

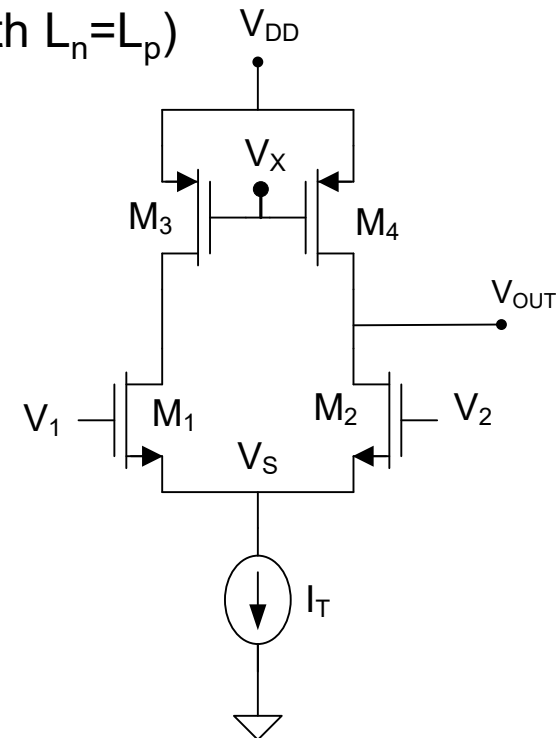
$$\sigma_{V_{os}}^2 \cong \frac{2}{W_n L_n} \left[ A_{VTO n}^2 + \frac{\mu_p}{\mu_n} A_{VTO p}^2 \right]$$

a)

$$\sigma_{V_{os}}^2 \cong \frac{2}{(0.5\mu)^2} \left[ .021^2 + \frac{1}{3} .025^2 \right]$$

$$\sigma_{V_{os}} \cong 72\text{mV}$$

$$3 \sigma_{V_{os}} \cong 216\text{mV}$$



Note this is a very large offset voltage !

# Random Offset Voltages

Example: Determine the  $3\sigma$  value of the input offset voltage for the MOS differential amplifier due to local random variations if:

- a)  $M_1$  and  $M_3$  are minimum-sized and
- b) the area of  $M_1$  and  $M_3$  are 100 times minimum size

$$\sigma_{V_{os}}^2 \cong 2 \left[ \frac{A_{VTO n}^2 + \frac{\mu_p L_n}{\mu_n W_n L_p^2} A_{VTO p}^2}{W_n L_n} \right]$$

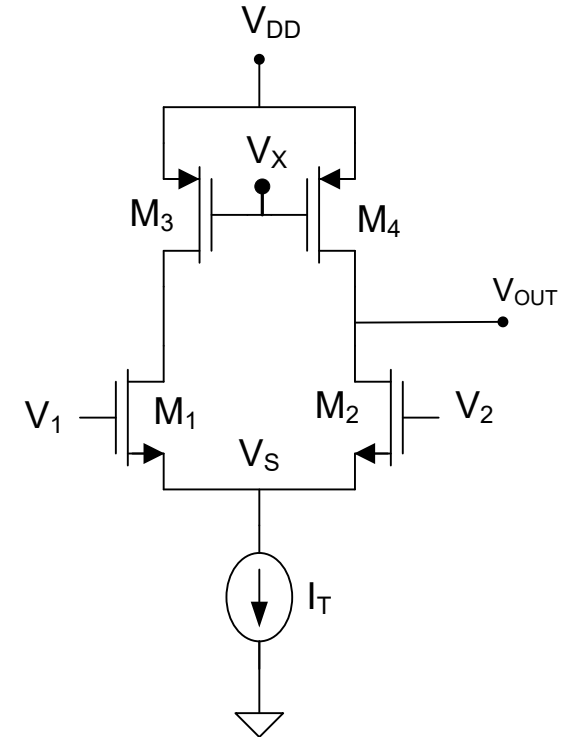
$$\sigma_{V_{os}}^2 \cong \frac{2}{W_n L_n} \left[ A_{VTO n}^2 + \frac{\mu_p}{\mu_n} A_{VTO p}^2 \right]$$

b)

$$\sigma_{V_{os}}^2 \cong \frac{2}{100(0.5\mu)^2} \left[ .021^2 + \frac{1}{3} .025^2 \right]$$

$$\sigma_{V_{os}} \cong 7.2\text{mV}$$

$$3\sigma_{V_{os}} \cong 21.6\text{mV}$$

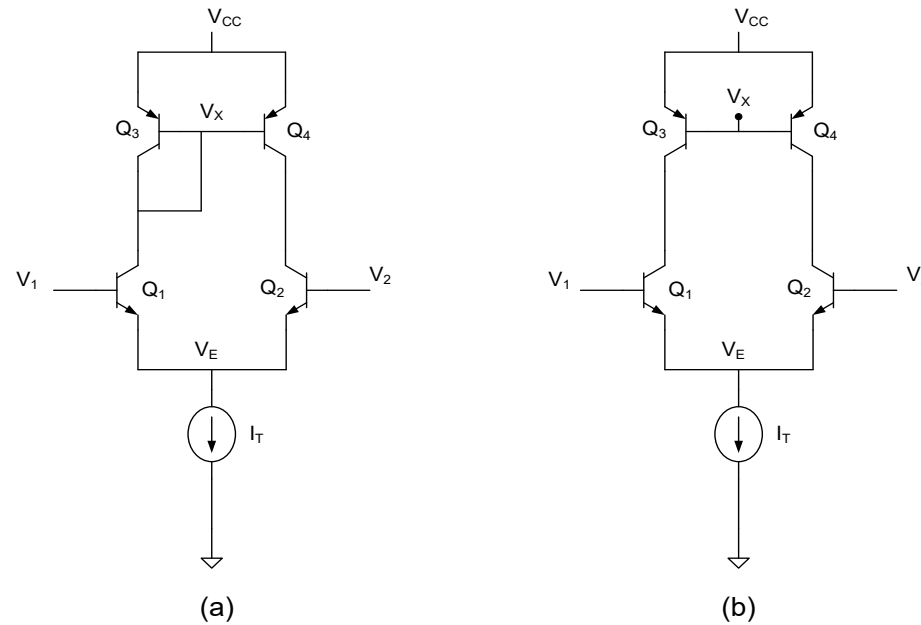


Note this is much lower but still a large offset voltage !

The area of  $M_1$  and  $M_3$  need to be very large to achieve a low offset voltage



# Random Offset Voltages



It can be shown that

$$\sigma_{V_{os}}^2 \cong 2V_t^2 \left[ \frac{A_{Jn}^2}{A_{En}} + \frac{A_{Jp}^2}{A_{Ep}} \right]$$

where very approximately

$$A_{Jn} = A_{Jp} = 0.1\mu$$

# Random Offset Voltages

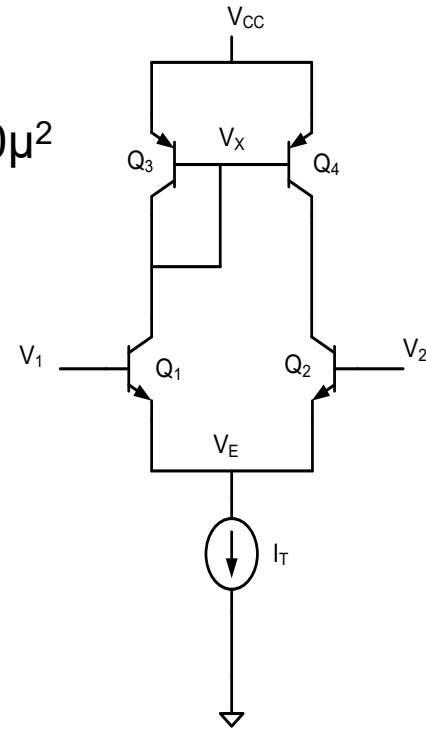
Example: Determine the  $3\sigma$  value of the offset voltage of a the bipolar input stage due to local random variations if  $A_{E1}=A_{E3}=10\mu^2$

$$\sigma_{V_{OS}}^2 \cong 2V_t^2 \left[ \frac{A_{Jn}^2}{A_{En}} + \frac{A_{Jp}^2}{A_{Ep}} \right]$$

$$\sigma_{V_{OS}} \cong \sqrt{2}V_t A_J \frac{\sqrt{2}}{\sqrt{A_E}}$$

$$\sigma_{V_{OS}} \cong 2 \cdot 25\text{mV} \cdot 0.1\mu \cdot \frac{1}{\sqrt{10\mu^2}} = 1.6\text{mV}$$

$$3\sigma_{V_{OS}} \cong 4.7\text{mV}$$



Note this value is much smaller than that for the MOS input structure !

# Random Offset Voltages

Typical offset voltages:

MOS - 5mV to 50mV

BJT - 0.5mV to 5mV

These can be scaled with extreme device dimensions

Often more practical to include offset-compensation circuitry



Stay Safe and Stay Healthy !

End of Lecture 22